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ABSTRACT

Presented is a discussion on problem solving in general and a description of a specific program, the Lane County Mathematics Project, to teach problem solving. Topics considered include: (1) Why teach problem solving; (2) Can problem solving be taught; (3) What are problem-solving skills; (4) What are some examples of problem-solving strategies; (5) How will these problem-solving materials fit into the regular mathematics program; (6) What in-service education is planned for teachers who use the packets; and (7) How will the programs be evaluated? A content outline for a fourth-grade packet is also presented. (MK)

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INTRODUCTION TO THE LCMP MATHEMATICS PROBLEM-SOLVING PROGRAMS*

by Oscar F. Schaaf

WHY TEACH PROBLEM SOLVING?

Problem solving is an ability people need throughout life. Pupils have many problems with varying degrees of complexity. Problems arise as they attempt to understand concepts, see relationships, acquire skills and get along with their peers, parents, and teachers. Adults have problems, many of which are associated with making a living, coping with the energy crisis, living in a nation with peoples from different cultural backgrounds, and preserving the environment. Since problems are so central to living, educators need to be concerned about the growth their pupils make in tackling problems.

CAN PROBLEM SOLVING BE TAUGHT IN MATHEMATICS?

Problem solving has been a primary stated goal of mathematics instruction since the turn of the century. It has been the subject of research by mathematics educators, educational psychologists and philosophers since the 1930's. Yet, an examination of the results of national tests, including the recent mathematics tests of the National Assessment of Educational Progress, indicates that students and graduates from our schools have serious deficiencies in their problem-solving abilities. Also, an examination of current mathematics texts and the results of classroom visitations reveals that problem solving is now only a minor part of mathematics instruction in both elementary and secondary schools.

Conjecture

All learners can improve their ability to use problem-solving skills if given the opportunity and motivation to do so. Educators can teach problem solving in the area of mathematics if

- they are provided with suitable materials for the pupils,
- they have developed an expertise in teaching the problem-solving process,
- a problem-solving approach to the teaching of mathematics is used frequently and regularly over a three-year period.

The purpose of this Oregon ESEA Title IV-C project is to test this conjecture after developing packets of materials for pupils and teachers and developing an inservice education model for teachers.

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* The federal government through the Oregon Department of Education and the Lane Education Service District provides the funds for this study. The proposal for the study was submitted by the staff of the Lane County Mathematics Project (LCMP).

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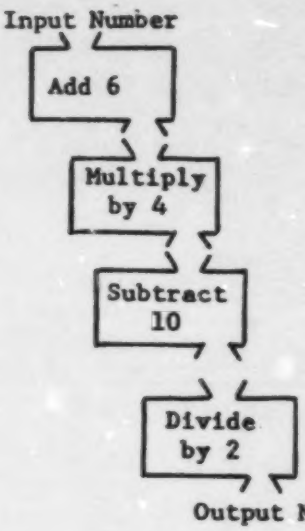
Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

A PROBLEM: WHAT IS IT?

Machine Hook-Ups



	Input Number	Output Number
a	4	
b	8	
c	12	
d		39
e		47
f		61

Suppose a 6th-grader were asked to fill in the missing output blanks for a, b, and c in the table. Would this be a problem for him? Probably not, since all he would need to do is to follow the directions. Suppose a 2nd-year algebra student were asked to fill in the missing input blanks for d. Would this be a problem? Probably not, since she would write the suggested equation,

$$\frac{4(x + 6) - 10)}{2} = 39$$

and then solve it for the input. Now suppose the 6th-grader were asked to fill in the input for d, would this be a problem for him? Probably it would be. He has no directions for getting the answer. However, if he has the desire, it is within his power to find the answer. What might he do? Here are some possibilities:

1. He might make guesses, do checking, and then make refinements until he gets the answer.
2. He might fill in the output numbers that correspond to the input numbers for a, b, and c.

	Input	Output
a	4	15
b	8	23
c	12	31
d		39

and then observe this pattern:

For an increase of 4 for the input, the output is increased by 8 .

Such an observation should lead quickly to the required input of 16.

3. He might start with the output and work backwards through the machine hook-up using the inverse (or opposite) operations.

For this pupil, there was no "ready-made" way for him to find the answer, but most motivated 6th-grade pupils would find a way.

A problem, then, is a situation in which an individual or group accepts the challenge of performing a task for which there is no immediately obvious way to determine a solution. Frequently, the problem can be approached in many ways. Occasionally, the resulting investigations are non-productive. Sometimes they are so productive as to lead to many different solutions or suggest more problems than they solve.

WHAT DOES PROBLEM SOLVING INVOLVE?

Problem solving requires the use of many skills. Usually these skills need to be used in certain combinations before a problem is solved. A combination of skills used in working toward the solution of a problem shall be referred to as a strategy. A successful strategy requires the individual or group to generate the information needed for solving the problem. A considerable amount of creativity can be involved in generating this information. Thus, problem solving, strategies, and creativity are closely related.

WHAT ARE PROBLEM-SOLVING SKILLS?

Skills are the building blocks used in solving a problem. The pupil materials in this packet afford many opportunities to emphasize problem-solving skills. A listing of some of these skills is given below. Some of the skills are unique to mathematics; others are interdisciplinary.

- Clarify the problem by asking questions.
- Follow directions.
- Visualize an object from its drawing or description.
- Select data which is needed for the solution of the problem.
- Share data and results.
- Listen to persons who have knowledge and experiences to share.
- Record.

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

- Measure.
- Make comparisons.
- Make and use a drawing or model.
- Make a systematic list or table.
- Break a problem into manageable parts.
- Use mathematics symbols to describe situations.
- Look for patterns and then make predictions.
- Make a reasonable estimate.
- Guess, check, and refine.
- Compute.
- Change a problem into one you can solve. Simplify the problem.
- Work backwards.
- Use what knowledge and skills you have.
- Detect and correct errors.
- Explain how you solved the problem.
- Find another answer when more than one is possible.
- Solve a problem by using different procedures.
- Create problems which can be solved by a given set of calculations.
- Create new problems by varying one that has been solved.

A list of problem-solving skills is helpful in reminding the teacher of important objectives to be considered. When an overview of problem solving is needed, such a list is cumbersome. On these occasions a list of categories of skills is more helpful. The categories used by the Lane County Mathematics Project are:

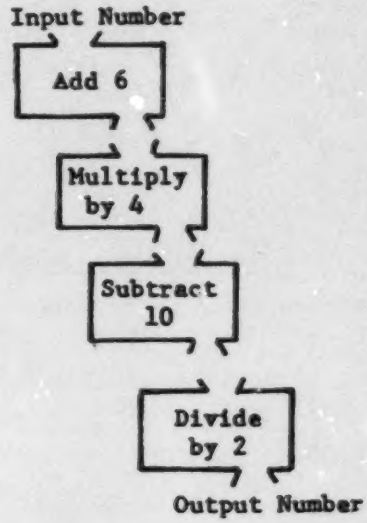
- Problem discovery, formulation
- Seeking information
- Analyzing information
- Solving--putting it together--synthesis
- Looking back - consolidating the gains
- Looking ahead - formulating new problems

The complete classified list of skills is included at the end of this commentary.

WHAT ARE SOME EXAMPLES OF PROBLEM-SOLVING STRATEGIES?

Since strategies are a combination of skills, a listing (if it were possible) would be even more cumbersome than the list of skills. Examples of some strategies that might be used in the Machine Hook-Ups problem (see pages 2 and 3) are:

1. Guess, check, and refine by observing the direction of error, check, possibly refine, check, ...
2. Make a systematic listing in a table, observe patterns, make prediction, check, possibly refine, check, ...
3. Use the knowledge you remember about the relationship between addition and subtraction and also multiplication and division, work backwards, check, ...



An awareness of the strategies being used to solve a problem is probably the most important step in the development of a pupil's problem-solving abilities.

HOW DOES PROBLEM SOLVING LOOK TO A PUPIL?

Up to this point we have been reviewing problem solving from the perspective of the teacher. John Mason[1] suggests we try also to view problem solving from the perspective of the pupil. He has identified problem-solving states which imply different pupil energy levels. What follows is an adaptation of his list with comments made in the first person.

Getting Started

I recognize that a problem exists.

Getting Involved

I accept the problem as one I will try.

Mulling

I am comfortable with the problem, am no longer completely lost, and am working at it.

[1]John Mason. "On Investigations," Mathematics Teaching. Sept. 1978

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

Keep Going

I am convinced that my efforts to solve the problem will help me to eventually solve it.

Insight

I get an idea and the end appears to be in sight.

Checking

I question whether my answer is really a solution.

Looking Back

I look at how I solved the problem. Can I use a similar way to solve a new or related problem?

The similarity between Mason's problem-solving states and the category of skills (see page 4) is certainly important to keep in mind as one teaches problem solving.

WHAT CAN BE DONE TO HELP A PUPIL BECOME A BETTER PROBLEM SOLVER? - A POINT OF VIEW

To increase the problem-solving skills of pupils:

A. CREATE A CLASSROOM ATMOSPHERE IN WHICH OPENNESS AND CREATIVITY CAN OCCUR.

Some things a teacher can do are:

- set an example by solving problems, some of which are within the intellectual grasp of the pupils and by sharing these experiences with them.
- regularly provide challenge problems for pupils to solve.
- reduce anxiety by encouraging communication and cooperation. On frequent occasions problems might be investigated using a cooperative mode of instruction along with brainstorm sessions.
- encourage pupils in their efforts to solve a problem by indicating that their strategies are worth trying and by providing them with sufficient time to investigate the problem; stress the value of the procedures pupils use.
- use pupils' ideas (including their mistakes) in solving problems and developing lessons.
- ask probing questions which make use of words and phrases such as

  I wonder if -
  Do you suppose that -
  What happens if -
  How could we find out -
  Is it possible that -

- reinforce the asking of probing questions by pupils as they search for increased understanding. Pupils seldom are skilled at asking probing questions but they can be taught to do so. If instruction is successful, questions of the type, "What should I do now?" will be addressed to themselves rather than to the teacher.

If these suggestions are used as guiding principles, then pupils could say as Johnson and Johnson[2] suggest that the classroom is

"... a place where we work together to learn and share our ideas: argue our points of view unless logically persuaded otherwise: help each other find the most appropriate answers: and sometimes we have fun competitions or work on our own, but most of the time we learn together."

B. TEACH PROBLEM-SOLVING SKILLS DIRECTLY WITH MANY OPPORTUNITIES FOR APPLICATION TO THE SOLUTION OF PROBLEMS

Teachers do emphasize computation and measurement. Other skills such as "follow directions," "listen," and "correct errors," are skills teachers expect pupils to master. Yet such skills as "guess and check," "make a systematic listing," or "change a problem into one you can solve," are seldom made the object of instruction. These skills, as well as many more, also need emphasis. Detailed examples for teaching these skills are given in the commentaries for the section entitled, Pre-Problem-Solving Activities.

C. PROVIDE MANY OPPORTUNITIES FOR PUPILS TO USE THEIR OWN PROBLEM-SOLVING STRATEGIES

Strategy formulation is the most crucial phase in solving problems. At this time the solver experiences uncertainty, surprise, confusion, delight, and frustration. In addition to creating an appropriate classroom atmosphere, teachers can help by relating their experiences at solving problems and by encouraging pupils to talk about the strategies they have used or are now using. Pupils need to feel they are making progress in their problem-solving efforts. Generally, instruction should be non-directive, but at times suggestions may need to be given. If possible, these suggestions should be made in the form of alternatives to be explored rather than hints to be followed.

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[2] Johnson, David W. and Johnson, Roger T. "Cooperative, Competive and Individualistic Learning," Journal of Research and Development in Education, Vol. 12. November 1, 1978.

One section of this packet includes a collection of challenge activities; in addition, others are scattered throughout other sections. These activities provide opportunities for emphasizing problem-solving strategies. A specific example on the teaching of these strategies is given in the commentary for the section entitled, Challenges, and in the teacher comments for each challenge.

D. INCORPORATE A PROBLEM-SOLVING EMPHASIS IN
- TEACHING MATHEMATICAL CONCEPTS, GENERALIZATIONS AND PROCESSES.
- DRILL AND PRACTICE ACTIVITIES.
- LABORATORY ACTIVITIES AND INVESTIGATIONS INVOLVING MATHEMATICAL APPLICATION AND READINESS ACTIVITIES.

If the approach to problem solving in mathematics consisted only of the inclusion of challege problems and the teaching of problem-solving skills, the program could result in the teaching of isolated problems without giving pupils a "feel" for mathematical relationships and a sense for the interconnectedness between topics from various branches of mathematics. To avoid this danger, the LCMP staff chose to include in each grade-level packet lessons for the teaching of topics common to the mathematics course of study for that level. The lessons also are designed to emphasize a variety of problem-solving skills and strategies.

HOW WILL THESE PROBLEM-SOLVING MATERIALS FIT INTO THE REGULAR MATHEMATICS PROGRAM?

As an example, the content for the Fifth Grade Packet for Program B is given below along with a brief overview. From this information you can decide whether the Packet fits your mathematics program. See page 15 for a similar outline for the Fourth Grade Packet Program A.

Section	Comments
I. Pre-Problem-Solving Activities	Six problems are included for the teaching of each of four different problem-solving skills. These are suggested for the beginning of the year to introduce pupils to problem solving. A direct mode of instruction is recommended.
II. Challenges	Fifteen challenges are included for use throughout the year. A non-directive instructional mode is recommended.

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

Section	Comments
III. Whole Number Drill and Practice	No sequence is implied by the order of activities included in this section. They can be used throughout the year but are especially appropriate at the beginning of the year when the initial chapter in the text emphasizes review.
IV. Story Problems	These lessons are excellent supplements for the story problems in your text since they emphasize problem-solving skills different from those usually found in texts.
V. Fractions	A circle model for fractions is used and the lessons should be done in sequence. We recommend that this section be used as a basis of your fraction teaching unit with the text being used in a supplementary fashion. It is best to work for the mastery of a region model like circles before doing much work with other models like sets or the number line. Operations with fractions are limited to a manipulative approach.
VI. Geometry	The lessons have few prerequisites and, therefore, can be done any time during the year. Most of these lessons deal with "global" properties of geometric objects often not covered in textbooks.
VII. Decimals	This decimal unit is developed without the use of fractions as a prerequisite. Decimals are introduced with a region model (a grid) followed by money and number line models. The lessons should be done in sequence and be used as the basis for your initial work with decimals with the text being used in a supplementary fashion.

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

Section	Comments
VIII. Probability (Experimental)	These laboratory-oriented lessons provide an opportunity to make and test conjectures. They also give an introduction to probability concepts. The lessons need not be done in sequence.
IX. Estimation with Calculators	The lessons need not be done in sequence. The calculator is used as a motivator for making estimates and doing mental arithmetic. If calculators are not available, many of these lessons still can be used.

ARE THERE TEACHER COMMENTARIES FOR THE GRADE-LEVEL PROBLEM-SOLVING PACKETS?

Commentaries are included for each lesson and section. The commentaries for each lesson usually include

- a list of mathematics teaching objectives.
- a list of problem-solving skills that could be emphasized.
- a list of materials needed.
- comments and suggestions for teaching the lesson and for emphasizing problem-solving skills and strategies.
- answers.
- problem extensions.

The commentaries for each section contain a brief rationale for the section and suggestions on using the activities in the classroom.

WHAT MATHEMATICS PROBLEM-SOLVING PROGRAMS ARE BEING DEVELOPED BY THE LANE COUNTY MATHEMATICS PROJECT? WHEN WILL THEY GENERALLY BE AVAILABLE TO OREGON SCHOOLS?

The LCMP staff has the task of preparing three programs. These are:

Program A - containing packets for elementary grades 4, 5, 6

Program B - containing packets for middle school grades 5, 6, 7

Program C - containing packets for junior high school grades 7, 8, 9.

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

The Programs are being field-tested according to the following schedule:

Program A		Program B		Program C	
Year	Packet	Year	Packet	Year	Packet
1979-80	Grade 4	1979-80	Grade 5	1980-81	Grade 7
1980-81	Grade 5	1980-81	Grade 6	1981-82	Grade 8
1981-82	Grade 6	1981-82	Grade 7	1982-83	Grade 9

Programs A and B should be ready for general dissemination in Oregon by September 1, 1982. Program C will be ready by September 1, 1983.

WHAT INSERVICE IS PLANNED FOR TEACHERS WHO USE THE PACKETS?

During the first year a consultant will provide six hours of planned inservice in the school (or district). In addition the consultants will provide on-call services as needed. The inservice is for all teachers who are or will be teaching the "field-test" materials during the three years of the field test. Teachers who will be field-testing the materials during the second and third years will be provided with some LCMP materials they can use with their pupils during the first year.

The major goals of the inservice are to acquaint teachers with problem solving itself and with teaching problem solving. Another goal is to prepare local teachers to assist in the consulting work in case other schools in the district or area choose to use these packets in subsequent years.

The six hours of planned inservice will be divided into several sessions. The schedule and activities for each session will be the product of planning by the consultants and teachers in the school. One three-session plan might be:

First Session - late August or early September $2\frac{1}{2}$ hours

Agenda:

1. Present this document ("An Introduction to the LCMP Problem-Solving Programs") with special emphasis on pages 6-8.
2. Show a video tape of a class presentation where problem-solving skills are emphasized.
3. Read the Teacher Commentary Pre-Problem Solving (pp i-vi) and decide upon a plan for teaching problem-solving skills.
4. Read Teacher Commentary II Challenges and compare it with the suggested approach for teaching problem-solving skills.

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

5. Present a problem from a Drill and Practice section for the teachers to solve collectively. The discussion which follows should include comments by teachers on the problem-solving skills they used.

Second Session - late September or early October - 2 hours

Agenda:
1. Conduct a problem-solving session for the teachers using problems from a Challenge section followed by a discussion of strategies used.
2. Show a videotape of a class solving a challenge problem.
3. Play an audiotape of a pupil who thinks aloud while solving a problem.
4. Present a plan for teaching problem-solving strategies followed by a discussion.
5. Give an overview along with special instructions for a section which presents new mathematical content.

Third Session - early January - $1\frac{1}{2}$ hours

Agenda:
1. Conduct a problem-solving session for teachers followed by a discussion of strategies used.
2. Serve as a convenor of a session in which teachers give accounts of their experiences teaching problem solving by using these materials and offer suggestions for improving the pupil and teacher packets.
3. Provide special instructions for the use of the sections to be used for the remainder of the school year.

Consultants will be thoroughly acquainted with the packets and with problem solving as a result of special instruction from the LCMP staff or through their own study and past experiences. Consultants will be those who are recognized leaders in mathematics education in the state of Oregon. Project staff members will keep in close communication with these consultants throughout the field-test period.

The inservice plan during the second and third field-test years will need to follow a similar plan for those teachers new to the Program. Teachers who will be field testing the packets during the second and third years will have a decided advantage over those doing it for the first year. They will have had their in-service and experience in using problem-solving materials before they become involved in the "official" field tests. The inservice plan and consultant services needed for the second and third year will be determined as the field test progresses. Possibly by the third year the consultant will need to provide the first session and then only be on an on-call basis.

WHAT IS THE PURPOSE OF THE FIELD TEST?

The pupil materials for each Packet, prior to the field test, have gone through two developmental stages, the commentaries through one, but the inservice plan has not been tested. The results from the field test should help us answer the question, How much teacher inservice is needed if a school chooses to use the packets?

Also, the field-test packets will be evaluated both from the formative and summative point of view. The results from the field test should help us answer these questions:

- Are these problem-solving packets effective in teaching problem solving?
- Are pupils who have three years instruction in problem solving better problem solvers than those who have not received this instruction?

HOW WILL THE PROGRAMS BE EVALUATED?

Four different aspects of problem solving will be evaluated, namely,

a. problem-solving skills
b. problem-solving preference
c. transfer of the process of problem solving
d. problem solving as measured by a standardized achievement test.

Tests of aspects a, b, and d will be given four times during the three-year period--before and after the first year, after the second year, and after the third year. A transfer instrument will be administered at the end of the third year. Some use will be made of control groups.

In addition to this summative evaluation, additional formative data will be collected for the purpose of revising the materials. Classroom visitations and pupil and teacher interviews will be made by the third-party evaluator. The field-test teachers also will fill out "record-of-use" forms.

With all this data, some judgment will be made concerning the effectiveness of the Programs.

Introduction to the LCMP Mathematics Problem-Solving Programs (cont.)

References

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Agency for Instructional Television. The Essential Learning Skills Television Project: Introduction to the Guide for THINKABOUT. Bloomington, Indiana, 1978

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Content Outline - Fourth Grade Packet Program A

Section	Comments
I. Pre-Problem - Solving Activities	Six activities are included for the teaching of each of four different problem-solving skills. These are suggested for the beginning of the year to introduce pupils to problem solving. A direct mode of instruction is recommended.
II. Challenges	Sixteen challenges are included for use throughout the year. A non-directive instructional mode is recommended.
III. Place Value Drill and Practice	No sequence is implied by the order of activities included in this section. They can be used throughout the year, but especially at the beginning of the year when place value receives a major emphasis.
IV. Whole Number Drill and Practice	The activities in this section are not sequenced and can be used in any order and at any time of the year after pupils have the prerequisites. The first five or six activities are especially suitable for the beginning of the year in conjunction with the text when addition and subtraction are reviewed.
V. Multiplication and Division Concepts and Rectangles	The <u>area</u> and <u>missing dimension</u> models are introduced for multiplication and division, respectively. This section is a prerequisite for section VIII, Two-Digit Multiplication. The lessons are developmental and should be done in sequence with the text being used in a supplementary fashion.

Content Outline - Fourth Grade Packet Program A (cont.)

VI. Fraction Concepts - Circular Regions	A circle model for fractions is used and the lessons should be done in sequence. We recommend that this section and section IX, Fractions and Mixed Number Concepts - Geoboards, be used as the basis for your work with fractions, with the text being used in a supplementary fashion. It is best to work for mastery of a region model (like circles or rectangles) before doing much work with other models like sets or the number line. Operations with fractions in this unit is limited to a manipulative approach.
VII. Geometry	These activities involve some exploration with tangrams and squares on a geoboard and then focuses mainly on area and perimeter concepts usually included in fourth grade texts. We recommend that this section be done after section V and before section IX.
VIII. Two-Digit Multiplication	These activities are developmental, expanding on the model used in section V, Multiplication and Division Concepts and Rectangles. The first lessons serve as a bridge between the place value model used in most basal texts for one-digit multiplication and the area model used here. Supplementary exercises from the class text or other sources will be needed to provide additional practice.
IX. Fractions and Mixed Number Concepts - Geoboards	In this section, proper and improper fractions and mixed numbers are represented on the geoboard with various rectangles being used as unit quantities. These activities are developmental, should be done in sequence and after section VI, Fraction Concepts - Circular Regions.

A CLASSIFIED LIST OF PROBLEM-SOLVING SKILLS

A. Problem Discovery, Formulation

1. State the problem in your own words.
2. Clarify the problem through careful reading and by asking questions.
3. Visualize an object from its drawing or description.
4. Follow written and/or oral directions.

B. Seeking Information

5. Collect data needed to solve the problem.
6. Share data and results with other persons.
7. Listen to persons who have relevant knowledge and experiences to share.
8. Search printed matter for needed information.
9. Make necessary measurements for obtaining a solution.
10. Record solution possibilities or attempts.
11. Recall and list related information and knowledge.

C. Analyzing Information

12. Eliminate extraneous information.
13. Find likenesses and differences and make comparisons.
14. Classify objects or concepts.
15. Make and use a drawing or model.
16. Make and/or use a systematic list or table.
17. Make and/or use a graph.
18. Look for patterns and/or properties.
19. Use mathematical symbols to describe situations.
20. Break a problem into parts.

D. Solve--putting it together--synthesis

21. Make predictions, conjectures, and/or generalizations based upon data.
22. Make decisions based upon data.
23. Make necessary computations needed for the solution.
24. Determine limits and/or eliminate possibilities.
25. Make reasonable estimates.
26. Guess, check, and refine.
27. Solve an easier but related problem. Study solution process for clues.

A Classified List of Problem-Solving Skills (cont.) -2-

D. Solve--putting it together--synthesis (cont.)

28. Change a problem into one you can solve.(Simplify the problem.)
29. Satisfy one condition at a time.
30. Look at problem situation from different points of view.
31. Reason from what you already know. (Deduce.)
32. Work backwards.
33. Check calculated answers by making approximations.
34. Detect and correct errors.
35. Make necessary measurements for checking a solution.
36. Identify problem situation in which a solution is not possible.
37. Revise the conditions of a problem so a solution is possible.

E. Looking Back - Consolidating Gains

38. Explain how you solved a problem.
39. Make explanations based upon data.
40. Solve a problem using a different method.
41. Find another answer when more than one is possible.
42. Double check solutions by using some formal reasoning method (mathematical proof).
43. Study the solution process.
44. Find or invent other problems which can be solved by certain solution procedures.
45. Generalize a problem solution so as to include other solutions.

F. Looking Ahead--Formulating New Problems

46. Create new problems by varying a given one.

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